

# The Weakest Failure Detectors to Boost Obstruction-Freedom

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**Abstract.** This paper determines necessary and sufficient conditions to implement *wait-free* and *non-blocking* contention managers in a shared memory system. The necessary conditions hold even when universal objects (like compare-and-swap) or random oracles are available, whereas the sufficient ones assume only registers.

We show that failure detector  $\diamond\mathcal{P}$  is the weakest to convert any obstruction-free algorithm into a wait-free one, and  $\Omega^*$ , a new failure detector which we introduce in this paper, and which is strictly weaker than  $\diamond\mathcal{P}$  but strictly stronger than  $\Omega$ , is the weakest to convert any obstruction-free algorithm into a non-blocking one.

## 1 Introduction

Multiprocessor systems are becoming more and more common nowadays. Multithreading thus becomes the norm and studying scalable and efficient synchronization methods is essential, for traditional locking-based techniques do not scale and may induce priority inversion, deadlock and fault-tolerance issues when a large number of threads is involved.

*Wait-free* synchronization algorithms [1] circumvent the issues of locking and guarantee individual progress even in presence of high contention. Wait-freedom is a liveness property which stipulates that every process completes every operation in a finite number of its own steps, regardless of the status of other processes, i.e., contending or even crashed. Ideal synchronization algorithms would ensure *linearizability* [2,3], a safety property which provides the illusion of instantaneous operation executions, together with wait-freedom.

Alternatively, a liveness property called *non-blockingness*<sup>4</sup> may be considered instead of wait-freedom. Non-blockingness guarantees global progress, i.e., that some process will complete an operation in a finite number of steps, regardless of the behavior of other processes. Non-blockingness is weaker than wait-freedom as it does not prevent some processes from starvation.

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<sup>4</sup> The term *non-blocking* is defined here in the traditional way [1]: “some process will complete its operation in a finite number of steps, regardless of the relative execution speeds of the processes.” This term is sometimes confused with the term *lock-free*. Note that non-blocking implementations provide a weaker liveness guarantee than wait-free implementations.

Wait-free and non-blocking algorithms are, however, notoriously difficult to design [4,5], especially with the practical goal to be fast in low contention scenarios, which are usually considered the most common in practice. An appealing principle to reduce this difficulty consists in separating two concerns of a synchronization algorithm: (1) ensuring linearizability with a minimal conditional progress guarantee, and (2) boosting progress. More specifically, the idea is to focus on algorithms that ensure linearizability together with a weak liveness property called *obstruction-freedom* [6], and then combine these algorithms with separate generic oracles that boost progress, called *contention managers* [7,8,9,10]. This separation lies at the heart of modern (obstruction-free) software transactional memory (STM) frameworks [7].

With obstruction-free (or OF, for short) algorithms, progress is ensured only for every process that executes in isolation for sufficiently long time. In presence of high contention, however, OF algorithms can livelock, preventing any process from terminating. Contention managers are used precisely to cope with such scenarios. When queried by a process executing an OF algorithm, a contention manager can delay the process for some time in order to boost the progress of other processes. The contention manager can neither share objects with the OF algorithm, nor return results on its behalf. If it did, the contention manager could peril the safety of the OF algorithm, hampering the overall separation of concerns principle.

In short, the goal of a contention manager is to provide processes with enough time without contention so that they can complete their operations. In its simplest form, a contention manager can be a randomized back-off protocol. More sophisticated contention management strategies have been experimented in practice [8,9,11]. Precisely because they are entirely devoted to progress, they can be combined or changed on the fly [10]. Most previous strategies were *pragmatic*, with no aim to provide *worst case guarantees*. In this paper we focus on contention managers that provide such guarantees. More specifically, we study contention managers that convert any OF algorithm into a non-blocking or wait-free one, and which we call, respectively, *non-blocking* or *wait-free* contention managers.

Two wait-free contention managers have recently been proposed [12,13]. Both rely on timing assumptions to detect processes that fail in the middle of their operations. This suggests that *some* information about failures might inherently be needed by any wait-free contention manager. But this is not entirely clear because, in principle, a contention manager could also use randomization to schedule processes, or even powerful synchronization primitives like compare-and-swap, which is known to be *universal*, i.e., able to wait-free implement any other object [1]. In the parlance of [14], we would like to determine whether a *failure detector* is actually needed to implement a contention manager with worst case guarantees, and if it is, what is the *weakest* one [15]. Besides the theoretical interest, determining the minimal conditions under which a contention manager can ensure certain guarantees is, we believe, of practical relevance, for this might help portability and optimization.

We show that the eventually perfect failure detector  $\diamond\mathcal{P}$  [14] is the weakest to implement a wait-free contention manager.<sup>5</sup> We also introduce a failure detector  $\Omega^*$ , which we show is the weakest to implement a non-blocking contention manager. Failure detector  $\Omega^*$  is strictly weaker than  $\diamond\mathcal{P}$ , and strictly stronger than failure detector  $\Omega$  [15], known to be the weakest to wait-free implement the (universal) consensus object [1].<sup>6</sup>

It might be surprising that  $\Omega$  is not sufficient to implement a wait-free or even a non-blocking contention manager. For example, the seminal Paxos algorithm [16] uses  $\Omega$  to transform an OF implementation of consensus into a wait-free one. Each process that is eventually elected a leader by  $\Omega$  is given enough time to run alone, reach a decision and communicate it to the others. This approach does not help, however, if we want to make sure that processes make progress regardless of the actual (possibly long-lived) object and its OF implementation. Intuitively, the leader elected by  $\Omega$  may have no operation to perform while other processes may livelock forever. Because a contention manager cannot make processes help each other, the output of  $\Omega$  is not sufficient: this is so even if randomized oracles or universal objects are available. Intuitively, wait-free contention managers need a failure detector that would take care of *every* non-crashed process with a pending operation so that the process can run alone for sufficiently long time. As for non-blocking contention managers, at least *one* process that never crashes, among the ones with pending operations, should be given enough time to run alone.

The paper is organized as follows. Section 2 presents our system model and formally defines wait-free and non-blocking contention managers. These definitions are, we believe, contributions in their own rights, for they capture precisely the interaction between a contention manager and an obstruction-free algorithm. In Sect. 3 and 4, we prove our weakest failure detector results. In each case, we first present (necessary part) a *reduction* algorithm [15] that *extracts* the output of failure detector  $\Omega^*$  (respectively  $\diamond\mathcal{P}$ ) using a non-blocking (respectively wait-free) contention manager implementation. When devising our reduction algorithms, we do not restrict what objects (or random oracles) can be used by the contention manager or the OF algorithm. Then (sufficient part), we present algorithms that implement the contention managers using the failure detectors and registers. These algorithms are devised with the sole purpose of proving our sufficiency claims. We do not seek to minimize the overhead of the interaction between the OF algorithm and the contention manager, nor do we discuss how the failure detector can itself be implemented with little synchrony assumptions and minimal overhead, unlike the transformations presented in [12]. However, as we show in [17], our algorithms can be easily extended to meet these challenges. The proofs of a few minor results are omitted due to the space limitations and can be found in the full version of the paper [18].

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<sup>5</sup>  $\diamond\mathcal{P}$  ensures that eventually: (1) every failure is detected by every correct (i.e., non-faulty) process and (2) there is no false detection.

<sup>6</sup>  $\Omega$  ensures that eventually all correct (i.e., non-faulty) processes elect the same correct process as their leader.

## 2 Preliminaries

**Processes and Failure Detectors.** We consider a set of  $n$  processes  $\Pi = \{p_1, \dots, p_n\}$  in a shared memory system [1,19]. A process executes the (possibly randomized) algorithm assigned to it, until the process *crashes (fails)* and stops executing any action. We assume the existence of a global discrete clock that is, however, inaccessible to the processes. We say that a process is *correct* if it never crashes. We say that process  $p_i$  is *alive* at time  $t$  if  $p_i$  has not crashed by time  $t$ .

A *failure detector* [14,15] is a distributed oracle that provides every process with some information about failures. The output of a failure detector depends only on which and when processes fail, and not on computations being performed by the processes. A process  $p_i$  queries a failure detector  $\mathcal{D}$  by accessing local variable  $\mathcal{D}$ -*output* <sub>$i$</sub> —the output of the module of  $\mathcal{D}$  at process  $p_i$ . Failure detectors can be partially ordered according to the amount of information about failures they provide. A failure detector  $\mathcal{D}$  is *weaker than a failure detector  $\mathcal{D}'$* , and we write  $\mathcal{D} \preceq \mathcal{D}'$ , if there exists an algorithm (called a *reduction* algorithm) that transforms  $\mathcal{D}'$  into  $\mathcal{D}$ . If  $\mathcal{D} \preceq \mathcal{D}'$  but  $\mathcal{D}' \not\preceq \mathcal{D}$ , we say that  $\mathcal{D}$  is *strictly weaker than  $\mathcal{D}'$* , and we write  $\mathcal{D} \prec \mathcal{D}'$ .

**Base and High-Level Objects.** Processes communicate by invoking primitive operations (which we will call *instructions*) on *base* shared objects and seek to implement the *operations* of a *high-level* shared object  $O$ . Object  $O$  is in turn used by an application, as a high-level inter-process communication mechanism. We call invocation and response events of a high-level operation  $op$  on the implemented object  $O$  *application events* and denote them by, respectively,  $inv(op)$  and  $ret(op)$  (or  $inv_i(op)$  and  $ret_i(op)$  at a process  $p_i$ ).

An *implementation* of  $O$  is a distributed algorithm that specifies, for every process  $p_i$  and every operation  $op$  of  $O$ , the sequences of *steps* that  $p_i$  should take in order to complete  $op$ . Process  $p_i$  *completes* operation  $op$  when  $p_i$  returns from  $op$ . Every process  $p_i$  may complete any number of operations but, at any point in time, at most one operation  $op$  can be *pending* (started and not yet completed) at  $p_i$ .

We consider implementations of  $O$  that combine a sub-protocol that ensures a minimal liveness property, called *obstruction-freedom*, with a sub-protocol that boosts this liveness guarantee. The former is called an *obstruction-free (OF)* algorithm  $A$  and the latter a *contention manager CM*. We focus on *linearizable* [2,3] implementations of  $O$ : every operation appears to the application as if it took effect instantaneously between its invocation and its return. An implementation of  $O$  involves two categories of steps executed by any process  $p_i$ : those (executed on behalf) of  $CM$  and those (executed on behalf) of  $A$ . In each step, a process  $p_i$  either executes an instruction on a base shared object or (in case  $p_i$  executes a step on behalf of  $CM$ ) queries a failure detector.

*Obstruction-freedom* [6,7] stipulates that if a process that invokes an operation  $op$  on object  $O$  and from some point in time executes steps of  $A$  alone<sup>7</sup>, then it eventually completes  $op$ . *Non-blockingness* stipulates that if some correct process never completes an invoked operation, then some other process completes infinitely many operations. *Wait-freedom* [1] ensures that every correct process that invokes an operation eventually returns from the operation.

**Interaction Between Modules.** OF algorithm  $A$ , executed by any process  $p_i$ , communicates with contention manager  $CM$  via calls  $try_i$  and  $resign_i$  implemented by  $CM$  (see Fig. 1). Process  $p_i$  invokes  $try_i$  just after  $p_i$  starts an operation, and also later (even several times before  $p_i$  completes the operation) to signal possible contention. Process  $p_i$  invokes  $resign_i$  just before returning from an operation, and always eventually returns from this call (or crashes). Both calls,  $try_i$  and  $resign_i$ , return *ok*.<sup>8</sup>

We denote by  $B(A)$  and  $B(CM)$  the sets of base shared objects, always *disjoint*, that can be possibly accessed by steps of, respectively,  $A$  and  $CM$ , in every execution, by every process. Calls  $try$  and  $resign$  are thus the only means by which  $A$  and  $CM$  interact. The events corresponding to invocations of, and responses from,  $try$  and  $resign$  are called *cm-events*. We denote by  $try_i^{inv}$  and  $resign_i^{inv}$  an invocation of call  $try_i$  and  $resign_i$ , respectively (at process  $p_i$ ), and by  $try_i^{ret}$  and  $resign_i^{ret}$ —the corresponding responses.

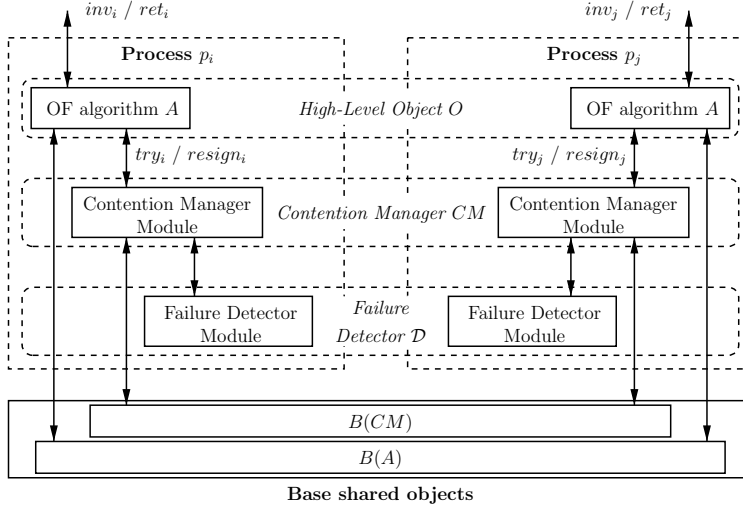
**Executions and Histories.** An *execution* of an OF algorithm  $A$  combined with a contention manager  $CM$  is a sequence of *events* that include steps of  $A$ , steps of  $CM$ , cm-events and application events. Every event in an execution is associated with a unique time at which the event took place. Every execution  $e$  induces a *history*  $H(e)$  that includes only application events (invocations and responses of high-level operations). The corresponding *CM-history*  $H_{CM}(e)$  is the subsequence of  $e$  containing only application events and cm-events of the execution, and the corresponding *OF-history*  $H_{OF}(e)$  is the subsequence of  $e$  containing only application events, cm-events, and steps of  $A$ . For a sequence  $s$  of events,  $s|i$  denotes the subsequence of  $s$  containing only events at process  $p_i$ .

We say that a process  $p_i$  is *blocked* at time  $t$  in an execution  $e$  if (1)  $p_i$  is alive at time  $t$ , and (2) the latest event in  $H_{CM}(e)|i$  that occurred before  $t$  is  $try_i^{inv}$  or  $resign_i^{inv}$ . A process  $p_i$  is *busy* at time  $t$  in  $e$  if (1)  $p_i$  is alive at time  $t$ , and (2) the latest event in  $H_{CM}(e)|i$  that occurred before  $t$  is  $try_i^{ret}$ . We say that a process  $p_i$  is *active* at  $t$  in  $e$  if  $p_i$  is either busy or blocked at time  $t$  in  $e$ . We say that a process  $p_i$  is *idle* at time  $t$  in  $e$  if  $p_i$  is not active at  $t$  in  $e$ .<sup>9</sup> A process *resigns* when it invokes *resign* on a contention manager.

<sup>7</sup> I.e., without encountering *step contention* [20].

<sup>8</sup> An example OF algorithm that uses this model of interaction with a contention manager is presented in [18]. A discussion about overhead of wait-free/non-blocking contention managers that explains when calls to *try/resign* can be omitted for efficiency reasons can be found in [17].

<sup>9</sup> Note that every process that has crashed is permanently idle.



**Fig. 1.** The OF algorithm/contention manager interface

We say that  $p_i$  is *obstruction-free* in an interval  $[t, t']$  in an execution  $e$ , if  $p_i$  is the only process that takes steps of  $A$  in  $[t, t']$  in  $e$  and  $p_i$  is not blocked infinitely long in  $[t, t']$  (if  $t' = \infty$ ). We say that process  $p_i$  is *eventually obstruction-free* at time  $t$  in  $e$  if  $p_i$  is active at  $t$  or later and  $p_i$  either resigns after  $t$  or is obstruction-free in the interval  $[t', \infty)$  for some  $t' > t$ . Note that, since algorithm  $A$  is obstruction-free, if an active process  $p_i$  is eventually obstruction-free, then  $p_i$  eventually resigns and completes its operation.

**Well-Formed Executions.** We impose certain restrictions on the way an OF algorithm  $A$  and a contention manager  $CM$  interact. In particular, we assume that no process takes steps of  $A$  while being blocked by  $CM$  or idle, and no process takes infinitely many steps of  $A$  without calling  $CM$  infinitely many times. Further, a process must inform  $CM$  that an operation is completed by calling *resign* before returning the response to the application.

Formally, we assume that every execution  $e$  is *well-formed*, i.e.,  $H(e)$  is linearizable [2,3], and, for every process  $p_i$ , (1)  $H_{CM}(e)|i$  is a prefix of a sequence  $[op_1][op_2], \dots$ , where each  $[op_k]$  has the form  $inv_i(op_k), try_i^{inv}, try_i^{ret}, \dots, try_i^{inv}, try_i^{ret}, resign_i^{inv}, resign_i^{ret}, ret_i(op_k)$ ; (2) in  $H_{OF}(e)|i$ , no step of  $A$  is executed when  $p_i$  is blocked or idle, (3) in  $H_{OF}(e)|i$ ,  $inv_i$  can only be followed by  $try_i^{inv}$ , and  $ret_i$  can only be preceded by  $resign_i^{ret}$ ; (4) if  $p_i$  is busy at time  $t$  in  $e$ , then at some  $t' > t$ , process  $p_i$  is idle or blocked. The last condition implies that every busy process  $p_i$  eventually invokes  $try_i$  (and becomes blocked), resigns or crashes. Clearly, in a well-formed execution, every process goes through the following cyclical order of modes: *idle, active, idle, ...*, where each *active* period consists itself of a sequence *blocked, busy, blocked, ...*

**Non-blocking Contention Manager.** We say that a contention manager  $CM$  guarantees non-blockingness for an OF algorithm  $A$  if in each execution  $e$  of  $A$  combined with  $CM$  the following property is satisfied: if some correct process is active at a time  $t$ , then at some time  $t' > t$  some process resigns.

A *non-blocking contention manager* guarantees non-blockingness for every OF algorithm. Intuitively, this will happen if the contention manager allows at least one active process to be obstruction-free (and busy) for sufficiently long time, so that the process can complete its operation. More precisely, we say that a contention manager  $CM$  is *non-blocking* if, for every OF algorithm  $A$ , in every execution of  $A$  combined with  $CM$  the following property is ensured at every time  $t$ :

**Global Progress.** If some correct process is active at  $t$ , then some correct process is eventually obstruction-free at  $t$ .

We show in [18] that a contention manager  $CM$  guarantees non-blockingness for every OF algorithm if and only if  $CM$  is non-blocking.

**Wait-Free Contention Manager.** We say that a contention manager  $CM$  guarantees wait-freedom for an OF algorithm  $A$  if in every execution  $e$  of  $A$  combined with  $CM$ , the following property is satisfied: if a process  $p_i$  is active at a time  $t$ , then at some time  $t' > t$ ,  $p_i$  becomes idle. In other words, every operation executed by a correct process eventually returns.

A *wait-free contention manager* guarantees wait-freedom for every OF algorithm. Intuitively, this will happen if the contention manager makes sure that every correct active process is given “enough” time to complete its operation, regardless of how other processes behave. More precisely, a contention manager  $CM$  is *wait-free* if, for every OF algorithm  $A$ , in every execution of  $A$  combined with  $CM$ , the following property is ensured at every time  $t$ :<sup>10</sup>

**Fairness.** If a correct process  $p_i$  is active at  $t$ , then  $p_i$  is eventually obstruction-free at  $t$ .

We show in [18] that a contention manager  $CM$  guarantees wait-freedom for every OF algorithm if and only if  $CM$  is wait-free.

In the following, we seek to determine the *weakest* [15] failure detector  $\mathcal{D}$  to implement a non-blocking (resp. wait-free) contention manager  $CM$ . This means that (1)  $\mathcal{D}$  implements such a contention manager, i.e., there is an algorithm that implements  $CM$  using  $\mathcal{D}$ , and (2)  $\mathcal{D}$  is *necessary* to implement such a contention manager, i.e., if a failure detector  $\mathcal{D}'$  implements  $CM$ , then  $\mathcal{D} \preceq \mathcal{D}'$ . In our context, a reduction algorithm that transforms  $\mathcal{D}'$  into  $\mathcal{D}$  uses the  $\mathcal{D}'$ -based implementation of the corresponding contention manager as a “black box” and read-write registers.

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<sup>10</sup> This property is ensured by wait-free contention managers from the literature [12,13].

### 3 Non-blocking Contention Managers

Let  $S \subseteq \Pi$  be a non-empty set of processes. Failure detector  $\Omega_S$  outputs, at every process, an identifier of a process (called a *leader*), such that all correct processes *in*  $S$  eventually agree on the identifier of the same *correct* process *in*  $S$ .<sup>11</sup>

Failure detector  $\Omega^*$  is the composition  $\{\Omega_S\}_{S \subseteq \Pi, S \neq \emptyset}$ : at every process  $p_i$ ,  $\Omega^*$ -*output* <sub>$i$</sub>  is a tuple consisting of the outputs of failure detectors  $\Omega_S$ . We position  $\Omega^*$  in the hierarchy of failure detectors of [14] by showing in [18] that  $\Omega \prec \Omega^* \prec \diamond \mathcal{P}$ .

To show that  $\Omega^*$  is necessary to implement a non-blocking contention manager, it suffices to prove that, for every non-empty  $S \subseteq \Pi$ ,  $\Omega_S$  is necessary to implement a non-blocking contention manager. Let  $CM$  be a non-blocking contention manager using failure detector  $\mathcal{D}$ . We show that  $\Omega^* \preceq \mathcal{D}$  by presenting an algorithm  $T_{\mathcal{D} \rightarrow \Omega_S}$  (Algorithm 1) that, using  $CM$  and  $\mathcal{D}$ , emulates the output of  $\Omega_S$ .

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**Algorithm 1:** Extracting  $\Omega_S$  from a non-blocking contention manager (code for processes from set  $S$ ; others are permanently idle)

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**uses:**  $L$ —register  
**initially:**  $\Omega_S$ -*output* <sub>$i$</sub>   $\leftarrow p_i$ ,  $L \leftarrow$  some process in  $S$   
 Launch two parallel tasks:  $T_i$  and  $F_i$

1.1 **parallel task**  $F_i$   
 1.2  $\lfloor \Omega_S$ -*output* <sub>$i$</sub>   $\leftarrow L$

1.3 **parallel task**  $T_i$   
 1.4 **while** *true* **do**  
 1.5  $\lfloor$  issue *try* <sub>$i$</sub>  and wait until busy (i.e., until call *try* <sub>$i$</sub>  returns)  
 1.6  $\lfloor L \leftarrow p_i$  // announce yourself a leader

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The algorithm works as follows. Every process  $p_i \in S$  runs two parallel tasks  $T_i$  and  $F_i$ . In task  $T_i$ , process  $p_i$  periodically (1) gets blocked by  $CM$  after invoking *try* <sub>$i$</sub>  (line 1.5), and (2) once  $p_i$  gets busy again, announces itself a leader for set  $S$  by writing its id in  $L$  (line 1.6). In task  $F_i$ , process  $p_i$  periodically determines its leader by reading register  $L$  (line 1.2).<sup>12</sup>

Thus, no process ever resigns and every correct process in  $S$  is permanently active from some point in time. Intuitively, this signals a possible livelock to  $CM$  which has to eventually block all active processes except for one that should run obstruction-free for sufficiently long time. By Global Progress,  $CM$  cannot

<sup>11</sup>  $\Omega_S$  can be seen as a restriction of the eventual leader election failure detector  $\Omega$  [15] to processes in  $S$ . The definition of  $\Omega_S$  resembles the notion of  $\Gamma$ -accurate failure detectors introduced in [21]. Clearly,  $\Omega_\Pi$  is  $\Omega$ .

<sup>12</sup> If a process is blocked in one task, it continues executing steps in parallel tasks.

block *all* active processes forever and so if the elected process crashes (and so becomes idle), *CM* lets another active process run obstruction-free. Eventually, all correct processes in  $S$  agree on the same process in  $S$ . Processes outside  $S$  are permanently idle and permanently output their own ids: they do not access *CM*.

This approach contains a subtlety. To make sure that there is a time after which the same correct leader in  $S$  is permanently elected by the correct processes in  $S$ , we do not allow the elected leader to resign (the output of  $\Omega_S$  has to be eventually stable). This violates the assumption that processes using *CM* run an obstruction-free algorithm, and, thus, a priori, *CM* is not obliged to preserve Global Progress. However, as we show below, since *CM* does not “know” how much time a process executing an OF algorithm requires to complete its operation, *CM* has to provide some correct process with *unbounded* time to run in isolation.

**Theorem 1.** *Every non-blocking contention manager can be used to implement failure detector  $\Omega^*$ .*

*Proof.* Let  $S \subseteq \Pi$ ,  $S \neq \emptyset$  and consider any execution of Algorithm 1. If  $S$  contains no correct process, then  $\Omega_S$ -*output* <sub>$i$</sub>  (for every process  $p_i \in S$ ) trivially satisfies the property of  $\Omega_S$ . Now assume that there is a correct process in  $S$ . We claim that *CM* eventually lets exactly one correct process in  $S$  run obstruction-free while blocking forever all the other processes in  $S$ .

Suppose not. We obtain an execution in which every correct process in  $S$  is allowed to be obstruction-free only for bounded periods of time. But the *CM*-history of this execution corresponds to an execution of some OF algorithm  $A$  combined with *CM* in which no active process ever completes its operation because no active process ever obtains enough time to run in isolation. Thus, no active process is eventually obstruction-free in that execution. This contradicts the assumption that *CM* is non-blocking.

Therefore, there is a time after which exactly one correct process  $p_j \in S$  is periodically busy (others are blocked or idle forever) and, respectively, register  $L$  permanently stores the identifier of  $p_j$ . Thus, eventually, every correct process in  $S$  outputs  $p_j$ : the output of  $\Omega_S$  is extracted.  $\square$

We describe an implementation of a non-blocking contention manager using  $\Omega^*$  and registers in Algorithm 2 (we prove its correctness in [18]). The algorithm works as follows. All active processes, upon calling *try*, participate in the leader election mechanism using  $\Omega^*$  in lines 2.3–2.5. The active process  $p_i$  that is elected a leader returns from *try* and is (eventually) allowed to run obstruction-free until  $p_i$  resigns. Once  $p_i$  resigns, the processes elect another leader. Failure detector  $\Omega^*$  guarantees that if an active process is elected and crashes before resigning, another active process is eventually elected.

**Theorem 2.** *Algorithm 2 implements a non-blocking contention manager.*

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**Algorithm 2:** A non-blocking contention manager using  $\Omega^* = \{\Omega_S\}_{S \subseteq \Pi, S \neq \emptyset}$

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**uses:**  $T[1, \dots, n]$ —array of single-bit registers  
**initially:**  $T[1, \dots, n] \leftarrow false$

2.1 **upon**  $try_i$  **do**  
2.2    $T[i] \leftarrow true$   
2.3   **repeat**  
2.4     $S \leftarrow \{p_j \in \Pi \mid T[j] = true\}$   
2.5   **until**  $\Omega_S\text{-output}_i = p_i$

2.6 **upon**  $resign_i$  **do**  
2.7    $T[i] \leftarrow false$

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## 4 Wait-Free Contention Managers

We prove here that the weakest failure detector to implement a wait-free contention manager is  $\diamond\mathcal{P}$ . Failure detector  $\diamond\mathcal{P}$  [14] outputs, at each time and every process, a set of *suspected* processes. There is a time after which (1) every crashed process is permanently suspected by every correct process and (2) no correct process is ever suspected by any correct process.

We first consider a wait-free contention manager  $CM$  using a failure detector  $\mathcal{D}$ , and we exhibit a reduction algorithm  $T_{\mathcal{D} \rightarrow \diamond\mathcal{P}}$  (Algorithm 3) that, using  $CM$  and  $\mathcal{D}$ , emulates the output of  $\diamond\mathcal{P}$ .

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**Algorithm 3:** Extracting  $\diamond\mathcal{P}$  from a wait-free contention manager

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**uses:**  $R[1, \dots, n]$ —array of registers  
**initially:**  $\diamond\mathcal{P}\text{-output}_i \leftarrow \Pi - \{p_i\}$ ,  $k \leftarrow 0$ ,  $R[i] \leftarrow 0$   
Launch  $n(n-1)$  parallel instances of  $CM$ :  $C_{jk}$ ,  $j, k \in \{1, \dots, n\}$ ,  $j \neq k$   
Launch  $2n-1$  parallel tasks:  $T_{ij}$ ,  $T_{ji}$ ,  $j \in \{1, \dots, n\}$ ,  $i \neq j$ , and  $F_i$

3.1 **parallel task**  $F_i$   
3.2   **while**  $true$  **do**  $R[i] \leftarrow R[i] + 1$  // ‘‘heartbeat’’ signal

3.3 **parallel task**  $T_{ij}$ ,  $j = 1, \dots, i-1, i+1, \dots, n$   
3.4   **while**  $true$  **do**  
3.5      $x_j \leftarrow R[j]$   
3.6      $\diamond\mathcal{P}\text{-output}_i \leftarrow \diamond\mathcal{P}\text{-output}_i - \{p_j\}$  // stop suspecting  $p_j$   
3.7     issue  $try_i^{ij}$  (in  $C_{ij}$ ) and wait until busy  
3.8     issue  $resign_i^{ij}$  (in  $C_{ij}$ ) and wait until idle  
3.9      $\diamond\mathcal{P}\text{-output}_i \leftarrow \diamond\mathcal{P}\text{-output}_i \cup \{p_j\}$  // start suspecting  $p_j$   
3.10    wait until  $R[j] > x_j$  // wait until  $p_j$  takes a new step

3.11 **parallel task**  $T_{ji}$ ,  $j = 1, \dots, i-1, i+1, \dots, n$   
3.12   **while**  $true$  **do** issue  $try_i^{ji}$  (in  $C_{ji}$ ) and wait until busy

---

We run several instances of  $CM$ . These instances use disjoint sets of base shared objects and do not directly interact. Basically, in each instance, only two processes are active and all other processes are idle. One of the two processes, say  $p_j$ , gets active and never resigns thereafter, while the other, say  $p_i$ , permanently alternates between being active and idle. To  $CM$  it looks like  $p_j$  is always obstructed by  $p_i$ . Thus, to guarantee wait-freedom, the instance of  $CM$  has to eventually block  $p_i$  and let  $p_j$  run obstruction-free until  $p_j$  resigns or crashes. Therefore, when  $p_i$  is blocked,  $p_i$  can assume that  $p_j$  is alive and when  $p_i$  is busy,  $p_i$  can suspect  $p_j$  of having crashed, until  $p_i$  eventually observes  $p_j$ 's "heartbeat" signal, which  $p_j$  periodically broadcasts using a register. This ensures the properties of  $\diamond\mathcal{P}$  at process  $p_i$ , provided that  $p_j$  never resigns.

As in Sect. 3, we face the following issue. If  $p_j$  is correct,  $p_i$  will be eventually blocked forever and  $p_j$  will thus be eventually obstruction-free. Hence, in the corresponding execution, obstruction-freedom is violated, i.e., the execution cannot be produced by any OF algorithm combined with  $CM$ . One might argue then that  $CM$  is not obliged to preserve Fairness with respect to  $p_j$ . However, we show that, since  $CM$  does not "know" how much time a process executing an OF algorithm requires to complete its operation,  $CM$  has to provide  $p_j$  with *unbounded* time to run in isolation.

More precisely, the processes in Algorithm 3 run  $n(n-1)$  parallel instances of  $CM$ , denoted each  $CM_{jk}$ , where  $j, k \in \{1, \dots, n\}$ ,  $j \neq k$ . We denote the events that process  $p_i$  issues in instance  $CM_{jk}$  by  $try_i^{jk}$  and  $resign_i^{jk}$ . Besides, every process  $p_i$  runs  $2n-1$  parallel tasks:  $T_{ij}$ ,  $T_{ji}$ , where  $j \in \{1, \dots, n\}$ ,  $i \neq j$ , and  $F_i$ . Every task  $T_{ij}$  executed by  $p_i$  is responsible for detecting failures of process  $p_j$ . Every task  $T_{ji}$  executed by  $p_i$  is responsible for preventing  $p_j$  from falsely suspecting  $p_i$ . In task  $F_i$ ,  $p_i$  periodically writes ever-increasing "heartbeat" values in a shared register  $R[i]$ .

In every instance  $CM_{ij}$ , there can be only two active processes:  $p_i$  and  $p_j$ . Process  $p_i$  cyclically gets active (line 3.7) and resigns (line 3.8), and process  $p_j$  gets active once and keeps getting blocked (line 3.12). Each time before  $p_i$  gets active,  $p_i$  removes  $p_j$  from the list of suspected processes (line 3.6). Each time  $p_i$  stops being blocked,  $p_i$  starts suspecting  $p_j$  (line 3.9) and waits until  $p_i$  observes a "new" step of  $p_j$  (line 3.10). Once such a step of  $p_j$  is observed,  $p_i$  stops suspecting  $p_j$  and gets active again.

**Theorem 3.** *Every wait-free contention manager can be used to implement failure detector  $\diamond\mathcal{P}$ .*

*Proof.* Consider any execution  $e$  of  $T_{\mathcal{D} \rightarrow \diamond\mathcal{P}}$ , and let  $p_i$  be any correct process. We show that, in  $e$ ,  $\diamond\mathcal{P}$ - $output_i$  satisfies the properties of  $\diamond\mathcal{P}$ , i.e.,  $p_i$  eventually permanently suspects every non-correct process and stops suspecting every correct process. (Note that if a process  $p_i$  is not correct, then  $\diamond\mathcal{P}$ - $output_i$  trivially satisfies the properties of  $\diamond\mathcal{P}$ .)

Let  $p_j$  be any process distinct from  $p_i$ . Assume  $p_j$  is not correct. Thus  $p_i$  is the only correct active process in instance  $CM_{ij}$ . By the Fairness property of  $CM$ ,  $p_i$  is eventually obstruction-free every time  $p_i$  becomes active, and so  $p_i$

cannot be blocked infinitely long in line 3.7. Since there is a time after which  $p_j$  stops taking steps, eventually  $p_i$  starts suspecting  $p_j$  (line 3.9) and suspends in line 3.10, waiting until  $p_j$  takes a new step. Thus,  $p_i$  eventually suspects  $p_j$  forever.

Assume now that  $p_j$  is correct. We claim that  $p_i$  must eventually get permanently blocked so that  $p_j$  would run obstruction-free from some point in time forever. Suppose not. But then we obtain an execution in which  $p_i$  alternates between active and idle modes infinitely many times, and  $p_j$  stays active and runs obstruction-free only for bounded periods of time. But the CM-history of this execution could be produced by an execution  $e'$  of some OF algorithm combined with  $CM$  in which  $p_j$  never completes its operation because  $p_j$  never runs long enough in isolation. Thus, Fairness is violated in execution  $e'$  and this contradicts the assumption that  $CM$  is wait-free. Hence, eventually  $p_i$  gets permanently blocked in line 3.7. Since each time  $p_i$  is about to get blocked,  $p_i$  stops suspecting  $p_j$  in line 3.6, there is a time after which  $p_i$  never suspects  $p_j$ .

Thus, there is a time after which, if  $p_j$  is correct, then  $p_j$  stops being suspected by every correct process, and if  $p_j$  is non-correct, then every correct process permanently suspects  $p_j$ .  $\square$

We describe an implementation of a wait-free contention manager using  $\diamond\mathcal{P}$  and registers in Algorithm 4 (we prove its correctness in [18]). The algorithm relies on a (wait-free) primitive  $GetTimestamp()$  that generates unique, locally increasing timestamps and makes sure that if a process gets a timestamp  $ts$ , then no process can get timestamps lower than  $ts$  infinitely many times (this primitive can be implemented in an asynchronous system using read-write registers). The idea of the algorithm is the following. Every process  $p_i$  that gets active receives a timestamp in line 4.2 and announces the timestamp in register  $T[i]$ . Every active process that invokes  $try$  repeatedly runs a leader election mechanism (lines 4.3–4.6): the non-suspected (by  $\diamond\mathcal{P}$ ) process that announced the lowest (non- $\perp$ ) timestamp is elected a leader. If a process  $p_i$  is elected,  $p_i$  returns from  $try_i$  and becomes busy.  $\diamond\mathcal{P}$  guarantees that eventually the same correct active process is elected by all active processes. All other active processes stay blocked until the process resigns and resets its timestamp in line 4.8. The leader executes steps obstruction-free then. Since the leader runs an OF algorithm, the leader eventually resigns and resets its timestamp in line 4.8 so that another active process, which now has the lowest timestamp in  $T$ , can become a leader.

**Theorem 4.** *Algorithm 4 implements a wait-free contention manager.*

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**Algorithm 4:** A wait-free contention manager using  $\diamond\mathcal{P}$ 

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**uses:**  $T[1, \dots, N]$ —array of registers (other variables are local)

**initially:**  $T[1, \dots, N] \leftarrow \perp$

```
4.1 upon  $try_i$  do
4.2   if  $T[i] = \perp$  then  $T[i] \leftarrow GetTimestamp()$ 
4.3   repeat
4.4      $sact_i \leftarrow \{j \mid T[j] \neq \perp \wedge p_j \notin \diamond\mathcal{P}\text{-output}_i\}$ 
4.5      $leader_i \leftarrow \operatorname{argmin}_{j \in sact_i} T[j]$ 
4.6   until  $leader_i = i$ 

4.7 upon  $resign_i$  do
4.8    $T[i] \leftarrow \perp$ 
```

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